

Introduction to Algebraic Number Theory

Professor Victor Kolyvagin

The purpose of the course is to study basics of algebraic number theory. In particular, the course will provide ground for further more advanced study.

The central theme will be the theory of divisibility in rings of algebraic integers, i.e. theory of divisors, a far-reaching generalization of the main theorem of arithmetic about uniqueness of prime decomposition of a natural number.

One of the goals of algebraic number theory is solving Diophantine equations. This is one of its origins as well. Historically, Kummer's work on Fermat's equation $x^l + y^l = z^l$ over the cyclotomic field $Q(\zeta_l)$, generated over rational numbers by l -th roots of unity, was very important for development of the theory. We will study arithmetic of cyclotomic fields and Fermat's equations over them as motivation for development of the theory of algebraic numbers and as a nice example of how it all works.

One year algebra course is a recommended prerequisite. "Number Theory" by Borevich and Shafarevich will be a useful source.